9.6 Minimize the Surface Area of a Cylinder

Many products are packaged in cylindrical containers. Consider the food items on the shelves in a grocery store. You can buy fruits, vegetables, soups, dairy products, potato chips, fish, and beverages in cylindrical containers.

Tools
- construction paper
- ruler
- scissors
- tape

Investigate

How can you compare the surface areas of cylinders with the same volume?

Method 1: Build Models

Your task is to construct three different cylinders with a volume of 500 cm^3.

Work with a partner or in a small group.

1. Choose a radius measurement for your cylinder. Calculate the area of the base.
2. Using the formula V_{cylinder} = (area of base)(height), substitute the volume and the area of the base. Solve for the height.
3. Calculate the circumference of the base.
4. Construct the rectangle that forms the lateral surface area of the cylinder. The rectangle should have a length equal to the circumference you determined in step 3 and a width equal to the height you determined in step 2. Tape the rectangle to form the curved surface of the cylinder.

5. a) Calculate the area of the rectangle.
   b) Calculate the total surface area of the cylinder, including the base and the top.

6. Record the results for this cylinder in a table.

Cylinder	Radius (cm)	Base Area (cm^2)	Height (cm)	Surface Area (cm^2)
1				
2				
3				

7. Repeat steps 1 to 6 to create two different cylinders, each with a volume of 500 cm^3.

8. Compare the surface areas and dimensions of the cylinders. Choose the cylinder that has the least surface area. How does its height compare to its diameter?

9. **Reflect** Compare your results with those of other groups in the class. Describe the dimensions of the cylinder with the least surface area. Are these dimensions the optimal ones? Explain.

Method 2: Use a Spreadsheet

1. Use a spreadsheet to investigate the surface area of cylinders with different radii that have a volume of 500 cm^3. Start with a radius of 1 cm.

	A	B	C	D	E
1	Radius (cm)	Base Area (cm^2)	Volume (cm^3)	Height (cm)	Surface Area (cm^2)
2	1	=PI()*A2^2	500	=C2/B2	=2*B2+2*PI()*A2*D2
3	2	=PI()*A3^2	500	=C3/B3	=2*B3+2*PI()*A3*D3
4					

2. Use **Fill Down** to complete the spreadsheet. What is the whole-number radius value of the cylinder with the least volume? Try entering a radius value 0.1 cm greater than this value. Does the surface area decrease? If not, try a value 0.1 cm less. Continue investigating until the surface area is a minimum for the radius value in tenths of a centimetre.

3. What is the radius of the cylinder with minimum surface area? How does this compare to the height of this cylinder?

4. Change the value of the volume in the spreadsheet to investigate the dimensions of a cylinder with minimum surface area when the volume is 940 cm^2. How do the radius and height compare?

5. Repeat step 4 for a cylinder with a volume of 1360 cm^2.

6. **Reflect** Summarize your findings. Describe any relationship you notice between the radius and height of a cylinder with minimum surface area for a given volume.

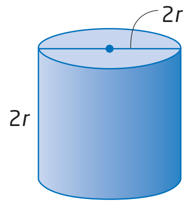
Example Minimize the Surface Area of a Cylinder

a) Determine the least amount of aluminum required to construct a cylindrical can with a 1-L capacity, to the nearest square centimetre.

b) Describe any assumptions you made.

Solution

a) For a given volume, the cylinder with minimum surface area has a height equal to its diameter.



The front view of this cylinder is a square.

Substitute $h = 2r$ into the formula for the volume of a cylinder.

$$\begin{aligned} V &= \pi r^2 h \\ &= \pi r^2(2r) \\ &= 2\pi r^3 \end{aligned}$$

Substitute the volume of 1 L, or 1000 cm^3, to find the dimensions of the cylinder.

$$1000 = 2\pi r^3$$

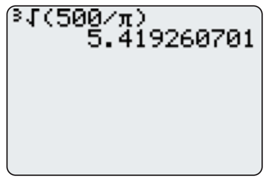
$$\frac{\overset{500}{\cancel{1000}}}{\underset{1}{\cancel{2}}\pi} = \frac{\overset{1}{\cancel{2\pi}}r^3}{\underset{1}{\cancel{2\pi}}}$$ **Divide both sides by 2π.**

$$\frac{500}{\pi} = r^3$$

$$\sqrt[3]{\frac{500}{\pi}} = r$$ **Take the cube root of both sides.**

$$5.42 \doteq r$$

C 500 ÷ π = $\sqrt[3]{\ }$



The radius of the can should be 5.42 cm. The height is twice this value, or 10.84 cm.

To find the amount of aluminum required, calculate the surface area.

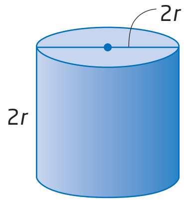
$$\begin{aligned} SA &= 2\pi r^2 + 2\pi rh \\ &= 2\pi(5.42)^2 + 2\pi(5.42)(10.84) \\ &\doteq 554 \end{aligned}$$

The least amount of aluminum required to make a cylindrical can that holds 1 L is about 554 cm^2.

b) The calculations in part a) do not take into account the extra aluminum required for the seam along the lateral surface. Also, along the top and bottom edges, there will likely be a rim that requires more aluminum.

Key Concepts

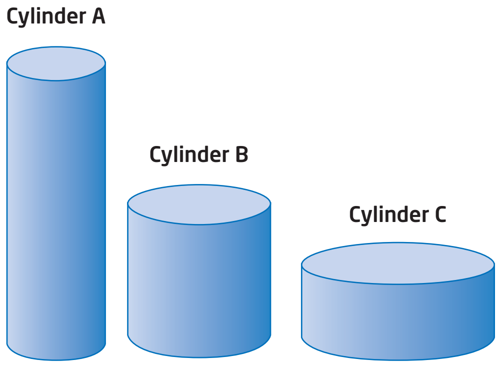
- For a cylinder with a given volume, a radius and a height exist that produce the minimum surface area.
- The minimum surface area for a given volume of a cylinder occurs when its height equals its diameter. That is, $h = d$ or $h = 2r$.
- The dimensions of the cylinder of minimum surface area for a given volume can be found by solving the formula $V = 2\pi r^3$ for r, and the height will be twice that value, or $2r$.



Communicate Your Understanding

C1 Describe a situation where it would be necessary to find the minimum surface area of a cylinder, given its volume.

C2 These cylinders all have the same volume. Which cylinder has the least surface area? Explain your answer.



Practise

For help with questions 1 to 3, see the Example.

1. Determine the dimensions of the cylinder with minimum surface area for each volume. Round the dimensions, to the nearest tenth of a unit.

a) 1200 cm^3 **b)** 1 m^3

c) 225 cm^3 **d)** 4 m^3

2. Determine the surface area of each cylinder in question 1 to the nearest square unit.

3. A cylindrical can is to have a volume of 540 cm^3. What should its dimensions be to minimize the amount of material used to make it? Round the dimensions to the nearest tenth of a centimetre.

Connect and Apply

4. A cylindrical gas tank is designed to hold 5 L of gas.

a) Determine the dimensions of the can that requires the least material. Round the dimensions to the nearest tenth of a centimetre.

b) Describe any assumptions you made in solving this problem.

5. Wade has been asked to design an insulated cylindrical container to transport hot beverages. To keep heat loss to a minimum, the total surface area must be minimized. Find the interior dimensions of the container with volume 12 L that has minimum heat loss. Round to the nearest tenth of a centimetre.

6. A cylindrical can must hold 375 mL of juice.

a) Determine the dimensions of the can that requires the least amount of aluminum. Round the dimensions to the nearest tenth of a centimetre.

b) If aluminum costs $0.001/cm^2, find the cost of the aluminum to make 12 cans.

7. Many of the cans found in our homes are not designed to use the least amount of material. Give reasons why the cans might be designed in other ways.

Did You Know?

The design of the USB is standardized by the USB Implementers Forum. The current specification is at version 2.0. This version supports three data-transfer rates: low speed, full speed, and high speed.

8. Chapter Problem Talia is shipping USB (universal serial bus) cables to a customer. She needs a container with a volume of 500 cm^3 that is as cost efficient as possible. Should she use a square-based prism box or a cylinder for the cables? Justify your answer mathematically.

9. A cylindrical building at Laurentian University in Sudbury, Ontario, is shown in the photo. Do you think it was designed to minimize the amount of heat loss? Justify your answer mathematically.

Achievement Check

10. Extra fuel tanks carried in the cabin of a plane are called ferry tanks. These tanks allow a plane to fly greater distances. A cylindrical ferry tank needs to hold 600 L of aircraft fuel.

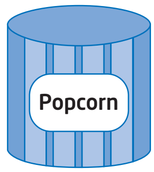
a) What are the dimensions of two possible cylindrical fuel tanks?

b) What should the dimensions of the tank be to minimize the amount of aluminum used in its construction?

c) How do these dimensions compare to the optimal square-based prism fuel tank?

Extend

11. A movie theatre sells popcorn in an open cylindrical container. The large size holds 1500 cm^3 of popcorn.

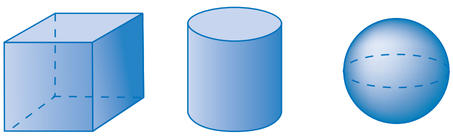


a) Determine the dimensions of the container that requires the least amount of cardboard.

b) How much cardboard is required to make one container?

c) Describe any assumptions you have made in solving this problem.

12. a) For a given volume, predict which three-dimensional figure will have the minimum surface area: a cube, a cylinder with height equal to diameter, or a sphere.



b) Check your prediction using the formulas for volume and surface area and a fixed volume of 1000 cm^3.

13. **Math Contest** You are to use 3584 cm^2 of newsprint. Determine the greatest volume that can be completely covered by the newsprint.

14. **Math Contest** Find the dimensions of the square-based prism box with maximum volume that can be enclosed in a cone with base radius 20 cm and height 30 cm.

15. **Math Contest** Find the dimensions that minimize the surface area for a cone with a volume of 225 cm^3.

16. **Math Contest** Find the dimensions of a cone with a surface area of 600 cm^2, if the cone has the greatest possible volume.